Curricular Redesign & Emerging Technology Grant (2009-2010)

Identification:

Project title: Developing elementary pre-service teachers’ TPACK in division of fractions by integrating SmartBoard, Mathematical Habits of Mind, and content knowledge in the design of instruction.

Principal investigator (Campus affiliation): Hsing-Wen Hu (UWRF)

Other significant persons: Mary-Alice Muraski (UWRF)
Scott Wojtanowski (UWRF)
Dan Reed (UWRF)
Emily Hogstrom (UWRF)

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Abstract:

Although previous studies have demonstrated pre-service elementary school teachers’ (PSTs) lack of understanding in division of fractions, little is known about their conceptual understanding toward this area. The extensive research on TPACK (Technological, pedagogical, and content knowledge) development model is used to explicate PSTs’ capacity on this integration and its effectiveness toward learning and teaching specific content such as division of fractions. A case study theoretical approach leads this study to look at PSTs’ TPACK development levels in a framework which facilitates description of their conceptions and their difficulties in this integrating process.

Project Narrative:

STATEMENT

Research in mathematics education has demonstrated that learning and teaching division of fractions has been one of the most problematic areas for elementary pre-service teachers (Ball, 1990; Schwartz, 2008; Tirosh, 2000), but little research has investigated the genuine reasons of such difficulties or revealed the alternative ways to understand the division of fractions. In many studies, researchers have gained significant insights in applying technology into mathematical learning and teaching (Harper, Schirack, Stohl, & Garofalo, 2001; Mishra, Koehler, 2006) and adopting Mathematical Habits of Mind into the mathematics curriculum (Goldenberg, Shteingold, Feurzeig, 2002), but researchers have yet to integrate technology (such as the SmartBoard) and Mathematical Habits of Mind into the curriculum for division of fraction. Without such analyses, mathematics education researchers have had to rely on less useful (consider rewording) inference when investigating pre-service teachers’ conceptual
understanding in division of fractions and how it might be improved. This case study examines teaching and learning division of fractions by creating a mathematical model for conceptual understanding in one elementary mathematics method course that used the TPACK. The main research questions were: (1) What were the primary TPACK levels before they participated in the workshop? (2) What progresses in TPACK levels did pre-service teachers have after they participated in the workshop? Constructing methods for answering such questions was an important component of the study.

The data we collected are based on observing one focus group in the workshop. We present our results in three sections—learning and teaching patterns in the observed workshop, model creations the pre-service teachers used to interpret division of fractions, and TPACK levels the pre-service teachers reached. We will use these analyses to illustrate the possibility of applying TPACK into mathematics methods courses which may help pre-service teachers learn and teach division of fractions more effectively and help them establish better evaluating strategies to assess students’ TPACK in division of fractions.

**METHODS**

*Participants*

The workshop participants, from a comprehensive state university in the mid-west, were enrolled in an elementary education initial certification program. This program is for individuals with an undergraduate degree in an area other than education, who wants to become a certified teacher in an elementary or middle school. Individuals also include those that wants to add elementary certification to a current teaching license. Fifteen students participated in this study. Two participants major in the sciences, and four students major in the social studies. Six students major in English and literature, and the rest major in the fine arts. Most of students have taken two math content courses before they got into the mathematics method course. There were eleven female participants and four male participants.

*Data collection*

Each pre-service teacher participated in six workshops within a two-month time period. All sessions were video-recorded and transcribed. In the deliberative workshop, each PST completed a common collection of tasks and also taught materials with conceptual understanding that is based on the principles of TPACK. With the customized TPACK teaching which integrated SmartBoard and Mathematical Habits of Mind into the content teaching, PST were encouraged to work on in-depth explanations and reflections. The information enabled me to make inferences about the conceptions of division of fractions underlying the observation and the explanation.

In effort to provide a description (a snapshot) of each PST’s currently held conceptions, I analyzed the first observation to get an initial understanding of the PST’s conceptions of TPACK in dealing with division of fraction and to identify areas for further investigation and workshop preparation. In order to test my initial model of PST’s conception, the second observation, which was in the sequences of workshop, was designed to explore those areas, clarify points, and
develop deeper conception in TPACK teaching. The final observation focused on PST’s holistic growth in teaching division of fraction with TPACK. I analyzed the transcripts of these observations to draft a more complete description about PSTs’ TPACK.

Participants have a forty-minute workshop with every lesson. Typically these observations for the focus group were completed with the other pre-service teachers in my regular method courses. This method allowed for more realistic observations. Furthermore, the observation descriptions were examined immediately after the observation was finished. This assisted me in finding meaningful ideas and insights for the next workshop.

For many pre-service teachers, facility integrating content knowledge (division of fractions), pedagogy knowledge (Mathematical Habits of Mind), and technology knowledge (SmartBoard) are unfamiliar to them. Therefore, instead of evaluating their work, I designed performance-based workshop activities to explore underlying phenomena and to develop PSTs’ capacities in TPACK. I consistently encouraged PSTs to invent more realistic strategies to improve their TPACK levels and to properly apply SmartBoard into the three functions—replacement, amplification, and transformation.

**Observation Tasks within the workshop**

The teaching content for the observation were developed by the researchers, piloted, and then modified. The workshops were designed to study the PSTs’ knowledge and capacity in performing TPACK in the following settings: reviewing previous knowledge for learning division of fractions, using SmartBoard to teach Type I & II division problems of fractions, introducing Mathematical Habits of Mind, using SmartBoard to teach Type III & IV division problems of fractions, extending SmartBoard’s function to transformation, samples reflections, and final project evaluation.

**Reviewing necessary knowledge for leaning division of fractions (02-25-10):**

At the beginning of workshop, all PSTs were asked to list ten concepts which are most important in learning division of fractions and then were asked to explain their reasons during group brainstorming. After groups agreed on concepts, the instructor used the “inspirations” program to organize their conceptions toward what previous concepts are needed for learning division of fractions. First, each group took turn contributing their ideas, while the instructor listed the ideas on the SmartBoard. Second, the instructor encouraged PSTs to group the lists on the board according to similarity. Third, based on the categories made, PSTs named each category. Once PSTs agreed on the names of the categories, the instructor asked PSTs to contribute other ideas into these categories. The categories included the basic concepts in fractions, addition and subtraction of fractions, multiplication of fractions, meaning of division, and advance concepts of fractions.

In order to help PSTs understand the division of fractions with conceptual understanding, this study adapted the common denominator algorithm to guide PSTs in reviewing and teaching of division of fractions. Two previous concepts were emphasized from the class’ categorizing. Concepts include the functions of common denominator and the meaning of fractions. The functions of common denominator can not only be used for dealing with adding and subtracting fractions with different denominators, but also be used for learners to learn division of fractions when the algorithm of “invert-multiply” can’t be understood. Understanding the meaning of
division had prepared learners to create models and to make more accurate mathematical sentences when they deal with division of fractions problems.

**Using SmartBoard to solve Type I & II division problems of fractions (03-03-10):**
Integrating SmartBoard into the processes of learning and teaching division of fractions became the main content in the second section of the workshop. PSTs were given type I and II fractional division problems. First, they needed to solve it with manipulatives. After that, they were encouraged to share their results by using the Smartboard. Based on the students’ work, the researcher not only learned of the PSTs’ TPACK conditions, but also found points to advise students’ learning and modified the demonstration after PSTs’ sharing. In the conclusion section, students were encouraged to find the pattern of type I & type II problems. For the assignment, participants needed to create a teaching activity which includes type I and II division problems and integrates the Smartboard into the teaching processes.

In the beginning of the next section, PSTs shared their work as a group. The instructor evaluated their work by the following items: 1) conceptual understanding: creating model and connection between representation; 2) SmartBoard functions as replacement, amplification, or transformation; 3) Mathematical Habits of Mind included in the teaching process or built up in the teaching process, even though it had not been introduced to them.

**Introducing Mathematical Habits of Mind & SmartBoard functioning as (03-11-10):**
In this section, the meaning of Mathematical Habits of Mind was first introduced to the PSTs. Then it was explained how Mathematical Habits of Mind is incorporated into the applications of the SmartBoard, so that it can be functioning as replacement, amplification, and transformation. The instructor first revealed eight Mathematical Habits of Mind with its examples to PSTs. Once they perceived what the Mathematical Habits Mind is, they were asked to create a mini lesson to teach Type I & II division problem of fractions by integrating Mathematical Habits of Mind into the teaching process. PSTs worked in groups. After they finished their project they shared their works with reasons of why they used certain kinds of Mathematical Habits of Mind.

During the workshop, the instructor invited a guest from technology service center to advise the class about the application of the SmartBoard. In order to extend the functions of the SmartBoard, SmartBoard functioning as replacement, amplification, and transformation was revealed to the PSTs. In addition, this study tried to reach the transformational instruction with the SmartBoard, Mathematical Habits of Mind, such as visualizing, experiencing, tinkering had been integrated into the PSTs curriculum design. This study hope PSTs would have the initiative to apply Mathematical Habits of Mind in lesson planning for teaching division of fractions.

**Using SmartBoard to teach Type III & IV division problems of fractions (04-01-10):**
In this section, PSTs were more confident in using the SmartBoard. The most challenging part is to create a model and explain the processes for type III and IV problems. In order to help PSTs overcome this challenge, there were several procedures for PSTs to follow: 1) Reviewing previous main topics or concepts such as the meaning of division, Type I & II problems, and Mathematical Habits of Mind; 2) Dealing with Type III and Type IV problem by having students create word problems, use manipulatives, create a model with its representations, and explain what has been done; 3) Making a choice for presentation. The procedures had two presenters come up to the front of the class. One displayed the work on the white board, and the other used
the SmartBoard to present the work; 4) PST compare the advantages and disadvantages for both ways of presentation; 5) Assigning directions and materials for practicing.

Sample presentations and reflective discussions (04-08-10):
In the beginning of this section, three participants were chosen to present their results by practicing through a mini lesson. The rest of students and the instructor evaluated their work by using the evaluative rubric. After they presented their work, the whole class had a reflective discussion based on their observations. Meantime, the observers also gave some suggestions for the presenters and the rubric we used. The feedback they gave became useful ideas for the researcher to work on the evaluation of the final project. Other than that, the instructor reaffirmed the three functions of the Smartboard—replacement, amplification, and transformation with more practical examples. This presentation expected PSTs to have more ideas toward transformation as they prepare for their final projects.

Final project evaluation (04-16-10 & 04-29-10):
There are two lesson sessions for evaluating participants’ final projects. Each participants need to choose one type of fractional division problems. After that they need to design a mini lesson related to TPACK. In other words, the project should show their change in capacity for integrating Mathematical Habits of Mind and SmartBoard into the content knowledge. Each student had 8 minutes to present their work. The instructor evaluated participants’ work based on the following aspects: 1) presenter’s conceptual understanding; 2) adequacy regarding applying Mathematical Habits of Mind; 3) techniques and skills in using SMART Math Tools; 4) incorporating the SmartBoard and Mathematical Habits of Mind into the teaching of fractional division.

Analysis

In analyzing the data, I focused on the PSTs’ TPACK for teaching division of fractions in a way that would enable learners to conceptualize the concepts with the way of conceptual understanding. Because little is known about the PSTs’ TPACK in learning and teaching division of fractions, I followed the methodology of interpretive-descriptive theory (McMillan & Schumacher, 1997).

Data analysis began as soon as data were collected and transcribed. I analyzed each case or event by carefully examining the PSTs’ actions, explanations, or illustrations in the contexts of the TPACK workshop. The following questions evaluate the PSTs’ TPACK levels and their performance in handling the curriculum related to the TPACK.

1. Did the PSTs illustrate or explain the different types of division problems of fractions well?
2. Did the PSTs make proper connections between the Smartboard and the principles of lesson design?
3. Did the PSTs attempt to adapt their lesson because they tried to integrate the SmartBoard into it?
4. Did the PSTs engage learners in exploration of division of fractions with the SmartBoard?
5. Did the PSTs use Mathematics Habits of Mind with the SmartBoard to engage students in conceptual understanding about the division of fractions?

By answering these questions, I developed an initial description of the PST’s conceptual understanding in division of fractions by observing their work within TPACK. In analyzing the observation transcriptions, I focused on the PSTs’ capacity of integrating Mathematical Habits of Mind and SmartBoard into the math curriculum as well as the PSTs’ conceptual understanding in the division of fractions before, during, and after workshops. When the focus group had been observed, their conceptual understanding and TPACK were compared and contrasted between before, during, and after workshops. After, I compared and contrasted other groups’ understanding and TPACK to continue in refining the previously unidentified conceptions.

I did not begin with the analysis of the PSTs’ conceptual understanding and TPACK statement with a theory but was guided by the five development levels (recognizing, accepting, adapting, exploring, and advancing) related to teaching with TPACK, because my aim was to uncover these situations. Thus, throughout the workshop, I constantly developed and assessed capacity of the PSTs’ conceptual understanding and TPACK application toward division of fractions. When several sections had been observed and transcribed, categorized about PSTs’ conceptual understanding and TPACK statement began to emerge. After completing all the observation sections, I reviewed and organized all these sections and refined initial categories until all sections fell within one of the established categories, leading to a framework for the PSTs’ conceptual understanding and TPACK statement.

Overall, the analysis was carried out through an iterative process in which I constructed and refined the categories through with the procedures of description, categorization, and connection. To establish intercoder reliability, an independent researcher coded all the data after the framework had been established and all observation sections had been categorized, agreeing with 83% of my original codes; the different in the coding were resolved through discussion.

RESULTS

To explain how the PSTs’ conceptual understanding and teaching performance in division of fractions was influenced by TPACK, a framework which was developed by Niess (2009) was applied to evaluate the influences through the data analysis. This framework is then used (a) to describe PSTs’ conceptual understanding in learning and teaching division of fractions when they created models and made connections between models and different representations within the processes of TPACK (b) to explain PST’s strengths and weaknesses in integrating MHOM and SmartBoard into the curriculum of division of fractions (c) to explain the PSTs’ performance in TPACK levels.

PSTs’ conceptual understanding in division of fractions

PSTs’ conceptual understanding in division of fractions can be categorized into two major items, and each major item has two sub-items: (a) creating models—includes various configures and convey concepts; (b) making connections between models and other different representations—models with symbols and models with language descriptions. The proceeding processes run
through the developing sequence within these categories. In order to genuinely perceive PSTs’ conceptual understanding, it was critical to investigate their situations regarding fundamental knowledge in division of fraction. Therefore, in this section, PSTs’ ways of solving division of fractions will be revealed before we look at PSTs’ conceptual understanding.

Evaluate PSTs’ previous knowledge for the division of fractions before workshop

A performance-based assessment had been conducted at the beginning of the workshop so that the researcher could perceive the PSTs’ strategies for solving the division problems of fractions. The activities of plan preparation or SmartBoard teaching in the beginning section of the workshop were focused at the stage. PSTs showed that they were familiar with the invert-multiply algorithm (or flip over and multiply). Some of the participants knew that they might be able to solve by using the common denominator. However, after they made the dividend and divisor as the same denominator, they still used the invert-multiply algorithm eventually to perform the work of solving division of fractions. Nevertheless, some PSTs had showed how they think of division of fractions with initial thinking of “conceptual understanding”:

S1  S1. This is ½ this is ¼. So ½ divide by ¼. So how many ¼ are in a ½. Which is 2. Does that make sense?
S2  You used a fraction plot. (laugh)
S3  Yeah. Hey! So we have 1 unit of 1 and divide that into units of ½. How many would you have?
   Oh that would be 2. So we have ½. How many ¼ can go into ½. That is kind of like a manipulative but it’s a pizza. Oh we are not split the ½ into 4 pieces we are splitting that ½ pizza into quarter pieces.

Even though these PSTs thought of division problems of fractions with the way of conceptual understanding, the mathematical sentence that showed their problem-solving processes was still not represented by the way of conceptual understanding but instead of the invert-multiply algorithm:

S1  So there is 2 of them in a ½. So if you said you have a ½ of a pizza.
S2  Note that is ½ into ¼ so you have to make that into ¼.
S1  It is saying ½ divide by how many quarter pieces.
S2  Which is 2?
S1  That is what showed up here when you flip around.

Disregarding the activity of designing mini lesson of integrating SmartBoard or presenting mini lesson in class, PSTs showed their understanding in the invert-multiply algorithm. Some the participants could describe the procedures with details:

S1  …so here we have ½ and we want to divide it into ¼. So here’s half of that cherry pie and 1/2 divided by ¼….So our question is ½ divided by ¼. 
S2  You take the first number ½ and switch second number so 4 over 1.
S1  And then you multiply ½ times 4/1. Can anyone tell me what you would get. What is 1 times 4?
S2  4
S1  What’s 2 times 1?
S2  2
S1  Right so if you take ½ divided by ¼. We flipped ¼ to 4/1. And we multiplied ½ by 4/1. And 1
times 4 is 4. And 2 times 1 is 2. What is 4/2 we know that that can simplified. What can it be simplified to?

S2 2
S1 yea the answer is 2.

PSTs were skillful in solving mathematical sentences of division of fractions by using invert-multiply algorithm. However, this study found that they had a difficult time in explaining why there is a need to change sign, flip the divisor, and multiply. During the procedures of designing and presenting mini lesson, students showed their shortage regarding how to explain the reasons of inverting and multiplying. For example, PSTs tend to neglect the procedure that helps elementary students know why need to invert-multiply:

S1 …1 divide by 1/2 . flip ½. You get 2/1 which is 2. Next one ½ divide by ¼ flip it. You get 2. We can explain the flipping the divisor.
S2 Yeah but. We need to explain why you flip the divisor.
S1 Can we already say we did this in a previous lesson? (laugh)…

Some PSTs tried to give the reasonable explanation about the algorithm of invert-multiply by creating models, but they did not succeed:

S1 … 1 divide by ½ is 2. So what you are trying to say is you have 1 piece.
S1 …We could compare a whole pizza…
S2 Yeah.
S1 Take the whole pizza and divide into ½. Then that ½ divided into ¼. Which gives you?
S3 What I do not understand is how you explain the whole flipping thing. You know.
S2 Yeah we this is a sheet of paper. We cut it in half so we have 1 2 3 4 5 6 7 8 9 10 11 halves. So you need a full something. A full pizza and cut in half and how many do we have?
S1 2
S2 2 halves. So it would be 2 over 2.

From the conversation, we discovered that these PSTs might not have sufficient knowledge to create the model for the algorithm of invert-multiply. Furthermore, they lack the skills in making the connection between the model and the mathematical sentence.

The ways to deal with the division problems of fractions are related to their perspective in this content area. Therefore, this study was also interested in revealing PSTs’ previous knowledge or concepts which related to the division of fractions. At the beginning of the workshop, PSTs’ participation supplied rich information for us to evaluate their understanding in the meaning of division and the basic knowledge infractions. First of all, we found that most pre-service teachers are not familiar with the meaning of division. For example, when they solved the problem: 1 ÷ ½, the conversation went as:

S1 We can’t? What is the real way of doing this? 1 divide by ½ is 2. So what you are trying to say is you have 1 piece.
S2 (reading from book.)
No meaningful process was discussed. Instead, there were just descriptions about procedures. They did not mention what the total, size of the group, and group number is, and then based on the information, decide whether to use measurement division or partitive division. Secondly, PSTs’ concepts regarding fractions and fractional operations are not significantly clear. For example, few students could point out what the dividend and divisor are. In addition, some PSTs could not determine what grade division of fractions can be taught to.

Besides evaluating PSTs’ basic understanding in fraction and strategies for solving fractional problems, this study is also interested in PSTs’ conceptual understanding in division of fractions. The two elements, creating model and connection between representations, are checked when this study investigated PSTs’ conceptual understanding. In our early performance-based assessment, we found that PSTs teacher knew how to create model with circles to deliver their ideas for solving division problem of fractions. Most of them only knew how to use circles to represent their works. After they solve the problem with the model, they were asked to use a mathematical sentence to describe their work. This study found that there were no connection between PSTs’ models and their mathematical sentences. For example, the models they created involve repeated subtraction division, but their mathematical sentences showed invert-multiply.

Assessing PSTs’ conceptual understanding during the workshop:
The workshop in this study could be categorized into three sections. The main topics in the first section are: reviewing basic concepts in fraction, the meaning of fractions, and teaching Type I and II division problems with Smart Math Tools. The second section introduces Mathematical Habits of Mind and three functions of SmartBoard as the main content. In addition, how to integrate these matters into the curriculum of division of fractions had been demonstrated during this section. Finally, Type III and IV division problems with TPACK were performed in this third section. Besides the teaching content, there were reviewing quizzes, informative assessments, or outcome evaluations for the researchers to gain information regarding the participants’ growth and progress in each section.

In order to deal with division of fractions effectively, PSTs need to be familiar with the meaning of division. Therefore, two meanings of division, measurement division and partitive division, have been introduced to these PSTs at the beginning of the workshops. PSTs were asked to create a word problem (type I or type II) and solve it so that they could show their understanding toward this topic. In order to avoid PSTs not understanding what the problem asks for or forgetting the purpose of doing some of the procedures, such as the purpose of $\frac{3}{4}$ becoming $\frac{6}{8}$, PSTs were encouraged to clarify the meaning of the math problem they made.
After the instructor’s demonstration, 9 of 15 PSTs could solve the division problems of fractions by applying the meaning of division, especially measurement (repeated subtraction) division. For example, when $\frac{3}{4} \div \frac{3}{8} = \frac{6}{8} \div \frac{3}{8}$, one of PSTs used manipulatives to present the processes, and then used repeated subtraction to find two groups, which each group includes $\frac{3}{8}$. Four of 15 PSTs did not understand the meaning of division completely, especially in the operation of the model. One of PSTs forgot the meaning of division.

Before PSTs enter the second section about how to integrate Mathematical Habit of Mind and extend the function of SmartBoard, the researcher conducted a performance-based assessment to check PSTs’ understanding in the content taught in the workshop lessons. PSTs were willing to solve division problems of fractions with the algorithm of “common denominator.” For example, one of PSTs solved a problem with clear illustration:

S: Type I problem—I have 2 birthday cakes and I have to give each 2/8 of a piece of cake. How many friends can I invite to my party?

So we have 2 cakes and splitting each cake into eighths 1-16 pieces.
So the 16 we changed the 2 to a fraction to create a common denominator 8.
8 times 2 is where I got my 16.
16 divided by 8 divide by 2/8. So that’s 16/8 over 2/8 which is 16/2 equals 8.
Okay so the type II problem I have 1/2 of a snickers bar and I want to give away 1/8 to as many people as possible.
How many pieces can I give away.
So I have ½ divided by 1/8.
So I’ve changed my fractions to a common denominator.
So I have 4/8 divide 1/8. So 4 divide by 1 equals 4.
So 4 people can a piece of the candy bar.

Besides using the algorithm of common denominator, PSTs could make better connection between models and mathematical sentences. However, PSTs did not know more ways to create models. They are usually limited to a circle as a model.

Assessing PSTs’ final projects:
In the third section, PST had learned and practiced how to integrate Mathematical Habits of Mind into the teaching processes. At the same time, they were also working on extending the functions of SmartBoard from replacement, passing through amplification, and reaching to transformation. In addition, type III and IV division problems of fractions were also revealed to these PSTs. After conducting the instructor’s demonstration and the PSTs’ practice, a final project was assigned to the PSTs: to teach a mini lesson to the class. PSTs’ conceptual understanding was also observed during the teaching processes.

To the focus group, two students (Anna and Mary) had performed the mini lesson which integrated Mathematical Habits of Mind and SmartBoard into the content of learning type III and IV division problems. In this part, only PSTs’ conceptual understanding will be discussed. Based on our observations, PSTs had difficulty creating a word problem properly for either the
type III or IV problem. For example, the word problem Mary made confused her students. Her problem goes as follow:

Mary: … I did my problem to show students how you can do it a real world application setting. The problem I came up with was ¾ of the class showed up for class today and she is also the gym teacher. She wants to divide her gym class into 1/5 teams. How many 1/5 teams can she make out of the ¾ of the class.

Obviously, it is a division problem of fractions. However, it is difficult for elementary students to understand the problem with the meaning of division such as repeated subtraction division. Eventually, this PST could not confirm whether the answer should represent how many people or how many teams. This confusion shows this PST might not understand the meaning of division and its application completely. Another student, Anna, made a better word problem which is easier for students to understand by applying the meaning of division. The way which she presented the problem is:

Anna …Goldie lox and baby bear. They only have a ½ a bowl of porage left. They have bowls that will hold 2/5. Will each of them get a full bowl? That is what we are going to find out today. Goldie lox and baby bear will be able to eat 2/5 of ½ of a bowl. So I have helped you out a little bit today. I have given you your known. What are our known? We have a ½ a bowl of porage and 2/5 serving size. So what are our known?

From the descriptions, we see that Anna pointed out the key terms, such as “total,” “serving size,” and “unknown” which revealed the meaning of division (measurement division) for students to understand the word problem.

One of the crucial issues for division of fractions is to clarify what the unit is and how it help solve problems. For example, the mathematical sentence ¾ ÷ 1/5 is looking for how many times 1/5 is in ¾. It infers that 1/5 is a unit and ¾ is the total, and we want to find how many units are in the total (3/4). However, Mary’s presentation expresses that “unit” and “times” share the same meaning. It confused some of the PSTs.

Two students, Anna and Mary who were in the focus group had difficulty creating reasonable model for Type III and Type IV problem. Anna’s model, which showed two different kinds of pictures, were compared to represent the mathematical sentence ½ ÷ 2/5 (see figure 01). In Mary’s work, she made four circles and made three of four circle a darker color to represent ¾. On the other side, she made five circles and made one of them a darker color to represent 1/5. She used the models to illustrate ¾ ÷ 1/5. Mary’s model made more sense. However, it still could not decide what the unit was and how to use the unit to compare to the total. In addition, there was no adequate connection between Anna’s model and mathematical sentence.
PST’s strengths and weakness in integration of MHOM, SmartBoard, and curriculum

In order to investigate the integration, three sections were conducted in this study—pre-assessment, workshop, and final assessment. PSTs’ work in each section might not have included each theme, but they supplied a better way to look at how PSTs integrated the MHOM and SmartBoard into the curriculum.

Pre-assessment sessions:
In this section, there was no significant integration which included MHOM, SmartBoard, and curriculum content (division of fractions). PSTs’ work showed more interconnection between MHOM and curriculum content, but limited intersection between SmartBoard and MHOM or SmartBoard and curriculum content. For example, two habits of mind, visualizing and describing, were used quite often when PSTs conceptualized the content and organized the lesson. There was a conversation which showed interconnection between MHOM and content knowledge:

S2 …Is there a way you can flip it on there?
S1 Yep. 1 divide by 1/2. flip ½. You get 2/1 which is 2. Next one ½ divide by ¼ flip it. You get 2. We can explain the flipping the divisor.
S2 Yeah but. We need to explain why you flip the divisor…

In addition, PSTs also paid attention on creating “visualizing setting or procedure” for students to gain information through the visualization. Here is the one of their work:

S1 …Do you want to throw in a shape?
S2 So first we should have them visualize it right?
S1 Right
S2 And then explain the steps…afterwards? So maybe two more new slides.
There were limitations for PSTs to interconnection their work toward Smart Board because of shortage of their knowledge and capacity in using SmartBoard. At the very beginning of the workshop, PSTs were asked to design a mini lesson which integrated SmartBoard into the teaching processes. The purpose of this project is to examine PSTs’ knowledge and skills toward the SmartBoard. Based on our data, we found that Anna, who is an elementary teacher in a private school, knew how to perform the SmartBoard. The rest of PSTs had very little understanding with the SmartBoard and even resisted to use its function. Here is an example:

These PSTs showed sufficient basic knowledge and skills in working on some software such as Microsoft Offices and the internet. Even they use these programs to create figures and mathematical sentences to display some mathematical concepts. However, these took a longer time and gained inaccurate outcomes. Most PSTs were not familiar with the functions of SMART Math Tool at that point. Even though few of them knew some functions, they might not know how to apply them for their mini lesson. For example, Ben who was the one in the focus group, knew that there was a shape-division function in SmartBoard, but he did not use it to make an adequate model to teach division of fraction in his mini lesson. In addition, PSTs used SmartBoard as a replacement for the white board.

Workshop sessions:
Before the instructor demonstrated how to use SMART Math Tools to convey the concept of division of fraction, PSTs’ presentation toward this topic always applied SmartBoard as the white board, as we mentioned before “SmartBoard functioning as replacement.” For example, one PST illustrated the problem $3 \div \frac{3}{4}$. He did not use any available tools as recourses to support his illustration and explanation. He drew four circles first, and then divided each circle into four fourths. After that he spread out these fourths to become several groups (each group had three one-fourths). The figure 02 shows the work which he had done.
After demonstrating how to use the tools in SmartBoard, PSTs were able to create models effectively for teaching division of fractions. For example, Mary knew more about how to work on her lesson with SMART Math Tools. At this point, she already learned to use SmartBoard as amplification by using several tools to help her present math concepts effectively (Figure 03).

In the developing stage of the workshop, PSTs focused on integrating Mathematical Habits of Mind into curriculum of division of fractions and the performing processes of SmartBoard. Basically, most PSTs had been familiar with using SMART Math Tools to improve their teaching effectiveness. However, they did need to work out something to extend SmartBoard’s functions. In addition, new content topics such as Type III & IV also were brought into this stage.

As I mentioned before, most PSTs were familiar with using SMART Math Tool, especially in their lesson preparations. In other words, the materials or information for preparing the mini lesson were conducted from SMART Math Tools. However, their teaching performance during the presentation was close to traditional ways. There were no movable configures or materials for learners to tinker and experiment with. PSTs had created descriptions and figures to display how to solve the problems as the textbooks do (Figure 04). In sum, PSTs use SmartBoard as amplification to prepare their teaching materials, but they used SmartBoard as replacement when they presented their lesson during the class.
In this stage, PSTs were able to apply different Mathematical Habits of Mind in their teaching. Obviously, visualizing and describing had been used quite often when they design their mini lesson so that learners can develop the habits of visualizing and describing during the teaching processes. In addition, Mathematical Habits of Mind have been considered by PSTs to develop learners’ higher thinking and change learners’ learning routine when they are using SmartBoard. In other words, Mathematical Habits of Mind could be another way to extend SmartBoard’s function to transformation. However, based on their self-evaluation, they thought they had not reached that level yet.

At the end stage of the workshop, we focused on examining PSTs’ final projects and their reflections toward their projects. Two PSTs, Anna and Mary, will be samples for us to discuss how the final project went and what the outcomes for the workshop were. To the aspect of technological knowledge, Anna and Mary have showed their confidence and skills on performing SmartBoard. For example, Anna created cartoon pictures and interactive teaching processes on her mini lesson. Learners were impressed by her work (see Figure 05-a). Mary guided learners to explore some functions to highlight the points which she wanted to emphasize, such as coloring shapes to illustrate the fractional relationship between a group of numbers (see Figure 05-b). Basically, their work was more than replacement and belonged to amplification.
During the teaching processes, Anna and Mary integrated Mathematical Habits of Mind—tinkering and experimenting as well as visualizing and describing several activities. Here are some conversations which related to this issue:

I That’s a very good discussion. Ok, let us go to the next…the teacher use mathematical habits of mind to engage students in thinking about the division of fraction. What kind the uh, mathematical habits of mind have been used?

S1 I would say the visualizing

I ...why do you say that?

S1 Because…precise descriptions of the steps…with the mathematical sentence accompanied by the pictures and the symbols…

I Anything else?

S2 I just kind of have an issue with uh the SmartBoard and Mathematical Habits of Mind because I think some of them really are not…with the SmartBoard besides I think you’re designing a whole lesson to the SmartBoard, you’re forgetting the tinkering and the experimenting and the more hands-on and the…

I …But even in the SmartBoard you can use the tinkering. You can have the kids come up here. So still have the tinkering, so when you design your lesson, you can have your students to perform tinkering on the table or on the SmartBoard. You have to make a decision and look at which is better? Any other ideas? Katie…

S3 Well, I was just thinking the same, the visualization. Uhm, I think maybe a little bit just of the…cause they are coming to the board and they’re filling out the circles…like they were able to like…something like that…

In this study, the instructor supplied an opportunity for PSTs to reflect what they had done in their final projects. Most of our conversations are concentrated on how their TPACK activities reach the situation of SmartBoard functioning as transformation besides its performing ways and limitations. In order to let the readers understand what PSTs’ point of view regarding SmartBoard functioning as “transformation” when they incorporate MHOM, Tech, and Content, the researcher make a chart (Chart 01) to compare PSTs’ points and the definition of transformation.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>PSTs’ view Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SmartBoard as transformation may change students’ learning routines such as:</strong></td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>• Knowledge about performing SmatBoard&lt;br&gt;• Interactive activities&lt;br&gt;• Techniques of SmartBoard&lt;br&gt;• Some programs on the internet</td>
</tr>
<tr>
<td>Cognitive processes</td>
<td>• Emphasize the importance of practice&lt;br&gt;• Engaging in multiple dimensions when students interact to SmartBoard</td>
</tr>
<tr>
<td>Problem solving</td>
<td>?</td>
</tr>
<tr>
<td><strong>SmartBoard as transformation may change teachers’</strong></td>
<td></td>
</tr>
<tr>
<td>Instructional practices</td>
<td>• Performance demonstration will be more effective&lt;br&gt;• Teachers record their classroom work and pose them on the website, so ST can think and review after they go home&lt;br&gt;• Use other technology (such as ipod) to make connection</td>
</tr>
<tr>
<td>Instructional roles</td>
<td>• Teachers won’t need to stick at the location of the board.</td>
</tr>
</tbody>
</table>
From the Chart 01, we see PSTs’ thoughts about the improvement toward “SmartBoard functioning as transformation.” Obviously, PSTs had more ideas in aspect of learning content and instructional practices for improving their teaching on reaching transformation. They have few thoughts about how to reach the transformation in the learners’ cognitive processes and in the aspect of instructional roles. There no words about how to get to transformation by dealing with students’ skills of problem solving.

*PSTs’ performance in TPACK levels*

Along with the sections of the workshop, PSTs’ integration-decided process toward TPACK was evaluated by a five-stage sequential process—recognizing, accepting, adopting, exploring, and advancing. PSTs’ levels were decided by using evaluation rubric to observe PSTs’ teaching behaviors.

At the beginning sections of the workshop, PSTs were not able to use SmartBoard well, even though they recognize the alignment of the SmartBoard with math teaching, they did not have the capacity to integrate the SmartBoard in teaching. In addition, PSTs were passively engaged in activities that lead to a choice to adopt teaching mathematics with some functions of SmartBoard during the workshop. Therefore, this study found that PSTs’ TPACK fell to the level of recognizing, accepting, adapting at the beginning of the workshop.

At the developing sections of the workshop, instructors continually demonstrated how to use the functions of SMART Math Tools to deal with division problem of fractions. Meanwhile, they also have opportunities to practice these techniques which they learned from the workshop. In other words, they had known how to use SmartBoard, but the ability of integrating the SmartBoard into the teaching still needed to be developed. Therefore, while working on these sections, PSTs’ SmartBoard-decision was always fallen on the level of recognizing. In order to promote PSTs’ deciding level, the instructor created group performances and reflections that supplied opportunities for PSTs to engage in activities so that these ways lead to a choice to adopt teaching for division of fraction with the appropriate functions of SmartBoard. Even if they conducted these activities passively, they eventually reached the level of adapting.

At the conclusion of the workshop, PSTs were asked to work on a final project to design a mini lesson. This mini lesson needed to show the integration of Mathematical Habits of Mind, SmartBoard, and division problems of fractions. The evaluation began with peer review in the group that PSTs looked at peers’ work and evaluate it, especially deciding their TPACK levels. There were 9 PSTs at the recognizing level, 3 PTSs at the accepting level, and 3 at the adopting level. Right after peer review, PSTs were guided to work on performance-based teaching that integrates MHOM and SmartBoard into the lessons of division of fractions. According to our observation record, there were 11 PSTs actively integrated teaching with an adequate function of SmartBoard and four PSTs passively engaged in the activity. After teaching, PSTs were guided by the instructor to evaluate the results of decisions to integrate teaching with the functions of
SmartBoard. In this part, either in the small group or the whole group, PSTs showed that they’ve reached the level of advancing (see Chart 02).

Chart 02

<table>
<thead>
<tr>
<th></th>
<th>Pre-evaluation</th>
<th>Guided practices</th>
<th>Final reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognizing</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accepting</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adopting</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Exploring</td>
<td></td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Advancing</td>
<td></td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

DISCUSSION

To help this study understand PSTs’ learning and teaching, regarding division of fractions with the TPACK model, a framework is needed for us to review their growth. Niess (2009) and his colleagues revealed that the framework consisted of four elements—curriculum & assessment, learning, teaching, and access. Therefore, this study will discuss the findings based on this framework which will further extend our understanding for the body of knowledge of TPACK.

**Curriculum & Assessment**

The results of our study correspond to previous studies about PSTs’ understanding and capacity toward the division of fractions. Firstly, this study found that PSTs’ concepts regarding fractions and fractional operation were not significantly clear or stable (Triosh, 2000). This could explain why sometimes PSTs performed well when they dealt with division of fractions, but sometimes not. Secondly, PSTs preferred or only know to use algorithm of invert-multiplication to solve the division problems of fractions (Ball, 1990). Thirdly, PSTs had difficulty explaining why it is needed to change sign, invert the divisor, and multiply dividend and divisor when they used the algorithm of invert multiplication for solving division problems of fractions. The findings were similar with Schwartz’ findings (2008).

Most PSTs knew the procedural techniques to work on the problems of division of fractions. PSTs’ perceives regarding the procedures of division of fractions are based on their memories and not from conceptual understanding (Niemi, 1996). For example, when PSTs were given a word problem of division of fractions, most of them immediately wrote a mathematical sentence on the board and then solved the problem by using invert-multiply algorithm. When the instructor asked them to use manipulatives to solve the problem, many PSTs had difficulties, especially when working on Type III and IV problems. The cause of this problem could be that PSTs were not familiar with the meaning of division (Schwartz, 2008). For example, before the instructor revealed the meaning of division and its application, PSTs were confused with how a fraction could be divided by another fraction. This also corresponds to Ball’s study (1996) about PSTs’ misconception about division of fraction. However, using the repeated subtraction (measurement) division help PSTs deal with division problems of fractions. PSTs were able to perceive the meaning of problems and make better explanations with manipulatives or a model.
Besides introducing the meaning of division to PSTs, the algorithm of common denominator and the concept of “unit” are also crucial for PSTs working on division of fractions using the way of conceptual understanding. This study found that PSTs could easily solve type I and II division problems of fractions by just using common denominator algorithm without emphasizing the concept of “unit.” Nevertheless, PSTs who dealt with the type III and IV problems did need to point out both the algorithm of common denominator and the concept of “unit.” This study also found that PSTs might be confuse with their answers if they did not look the divisor as a unit and then use it to compare it to the dividend.

Assessing PSTs’ conceptual understanding toward the division of fraction, there are two categories to look at their capacity and growth. They are “create model” and “make connection between model and other representations” (Niemi, 1996). At the beginning, PSTs recognized that SmartBoard could be a teaching tool for them to make their lesson more interesting and formed a favorable attitude toward integrating SmartBoard to create a model for demonstration. However, they did not know how to start and perform the functions of SmartBoard adequately. Therefore, the instructor developed performance-based activities to help PSTs create a model for solving division of fractions. After looking at the instructor’s demonstration and being guided to practice, PSTs were able to create models and convey the concept for type I & II except to make various figures (models). To the type III and IV problems, PSTs had trouble using the concept of the “unit” to explain the answer after they made the dividend and the divisor with the same denominator. It implied that the models created through Smart Math Tools might not be able to enhance PSTs’ understanding of finding how many units (divisor) be could taken away from the total (dividend). In addition, this study also found that making connection between a model and other representations (such as symbolic or language descriptions) was the area which PSTs needed to improve. For example, PSTs did a better job creating a model and convey the target concept by using common denominator algorithm along with the meaning of division, but the mathematical sentence expressed the algorithm of invert-multiplication. Working on the SmartBoard still did not improve the situation. Our PSTs’ situation might be similar with the Ball’s study (1996) which our students have been taught only way—invert-multiplication to solve division of fraction. In addition, these PSTs had been taught with conceptual understanding to think and solve the division problems of fraction when they were in sixth grade (Niemi 1996). It was a challenge for them using the model to share their thinking about division of fractions if they did not learn well at that age.

To the aspect of curriculum & assessment, PSTs’ TPACK development was at a level of recognizing and accepting in the beginning. During the workshop, the development had been moved to the level of adapting which PSTs engaged in the workshop activities that lead them to adopt teaching and learning division of fractions with Smart Math Tools. At the end of the workshop, by evaluating their final projects, this study found that PSTs have reached the level of exploring where PSTs actively integrate teaching and learning teaching division of fractions with appropriate functions of the SmartBoard. In addition, a reflection session had been conducted
right after PSTs presented their work. PSTs’ TPACK development had reached the level of advancement where PSTs evaluate the results of their decision to integrations.

**Learning**

In this study, PSTs did not deal with matters that are related to children’s learning much. It might be because they were focusing on teaching the aspect or they were short of knowledge and skills regarding to children’s learning in division of fractions (Tirosh, 2000). At the beginning of the workshop, PSTs’ mini lessons fell on two situations: 1) Mathematical exploration related to division of fraction with SmartBoard were rarely seen; 2) Limited student SmartBoard use. This finding showed that PSTs TPACK development in the learning was at the level of recognizing and accepting before they received the training in the workshop. During the workshop, PSTs’ consideration toward the children’s learning in division of fractions was shown in the progress. For example, their lessons included activities which children explore different models for solving division of fractions by using Smart Math Tools. In addition, PSTs have already used a few of Mathematical Habits of Mind such as visualizing and describing to extend the functions of Smartboard. In this stage, PSTs’ TPACK development level toward the aspect of “learning” is adapting. At the end of the workshop, PSTs were familiar with using SmartBoard as a tool to facilitate them to guide children in learning division of fractions. For example, learning a new concept needed to pass through three developmental sequences: physical, pictorial, to symbolic. Some PSTs adequately incorporated physical and pictorial sequences as one when they created a model on the SmartBoard with its special functions. In addition, tinkering and experimenting, which are part of Mathematical Habits of Mind, were used for students to explore the division problems of fractions besides using visualizing and describing. At this stage, PSTs’ TPACK development had grown to the level of exploring.

**Teaching**

Using SmartBoard as a white board implies presenters have little knowledge on this technological equipment or they might be concerned that working on SmartBoard will take time away from teaching mathematics (Niess, 2006). At the beginning of this workshop, PSTs’ presentation toward teaching division of fractions tended to apply the SmartBoard as a white board. Based on our observation, our PSTs were not familiar with the functions of SMART Math Tool at that point. Even though few of them knew some functions, but they might not know how to apply them for their mini lesson because they have not been taught in previous courses. This might explain why PSTs showed sufficient basic knowledge and skills in working on some software such as Microsoft Offices and internet. However, they took a longer time and gained inaccurate outcomes when they used these programs to create figures and mathematical sentences to display some mathematical concepts. Based on the TPACK development model, we may say that at this stage, our PSTs were at the levels of recognizing and accepting.

After demonstrating how to use the tools in SmartBoard, PSTs were able to create models effectively for teaching division of fraction. Most PSTs were familiar with using SMART Math Tools to improve their teaching effectiveness, especially in their lesson preparations. In other
words, the materials or information for preparing the mini lessons were conducted from SMART Math Tools and were applied sufficiently by PSTs. However, PSTs’ teaching performance during the presentation was close to traditional ways. There were no movable configures or materials for learners to tinker and experiment with. PSTs created descriptions and figures to display how to solve the problems as the textbooks do. The reason for this issue might be related to PSTs’ abilities regarding to curriculum design, time limitation, or classroom environment. According to the TPACK development model, the work which PSTs presented here was set on the level of adapting.

In PSTs’ final project, they showed more confidence and skillful techniques on performing SmartBoard for teaching division of fractions. Their works were more than replacement and belong to amplification. At the reflection sections, PSTs could genuinely evaluate the results of the final project which integrate Mathematical Habits of Mind and SmartBoard into the curriculum of division of fractions. Therefore, they have reached the level of exploring and advancing.

Access
There were limitations for PSTs to interconnect their work with learners’ participation because of the shortage of their knowledge and capacity in using SmartBoard. For example, even though they recognize the alignment of the SmartBoard with leaners’ engagement, they did not have the capacity to create meaningful activities for learners to involve. Therefore, in the aspects of accessing SmartBoard, PSTs’ TPACK development fell into the level of recognizing and accepting. When they got into the developing stage of the workshop, most PSTs have been familiar with using SMART Math Tools. The interactive activities between learners and SmartBoard could be found at this stage. Therefore, PSTs had reached the level of exploring of TPACK development.

Exceptions to the original proposal:
The study was conducted with qualitative research methods. There were a lot of needs to observe and record the instructor’s and participants’ behaviors and works. After observation and recording, the data needed to be transcribed. Therefore, this study increased research assistants’ working hours. This matter caused deviations on our budget items.

Conclusion:
This study supplies a view for math educators using another way to teach division of fractions by applying TPACK development Model. According to the outcomes of this study, we found that integrating Mathematical Habits of Mind and the functions of the SmartBoard into the curriculum of division of fractions is a considerable way to help teachers or pre-service teachers organize their lessons or curriculum, so that they can guide their students to perceive mathematic concepts with conceptual understanding. Based on our findings, we confirm that Smart Math Tools is a powerful tool for teachers to create accurate and various models. The models that teachers make, whether it can conveys the key concepts or not, depends on the teachers’ previous knowledge, such as the meaning of division and the concept of the common denominator. This study also testifies that learning and teaching Mathematical Habits of Mind supply another direction to improve learners’ conceptualization. On the one hand, Mathematical
Habits of Mind promote the development of learners’ conceptual understanding by continually integrating habits such as patterning, describing, visualizing, tinkering, etc., into the problem-solving processes. On the other hand, these habits work together to help teachers use adequate tools on the SmartBoard, so the SmartBoard may properly function as a replacement, amplification, and transformation.