The Role of Definitions in Learning Geometry

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Introduction

Future middle and high school mathematics teachers must be able to give precise verbal definitions of geometric objects and concepts. In addition, they must be able to use proper logical deductions to write proofs and solve problems. Both definitions and proofs have been on my exams for a long time. I was surprised how little connections students make between the two kind of knowledge, so I designed an experiment to answer the following questions.

Research Questions

1. **Does verbalizing a related key concept before a proof help students solve the problem better?**

2. **Does knowing definitions better help students solve related proof problems?**

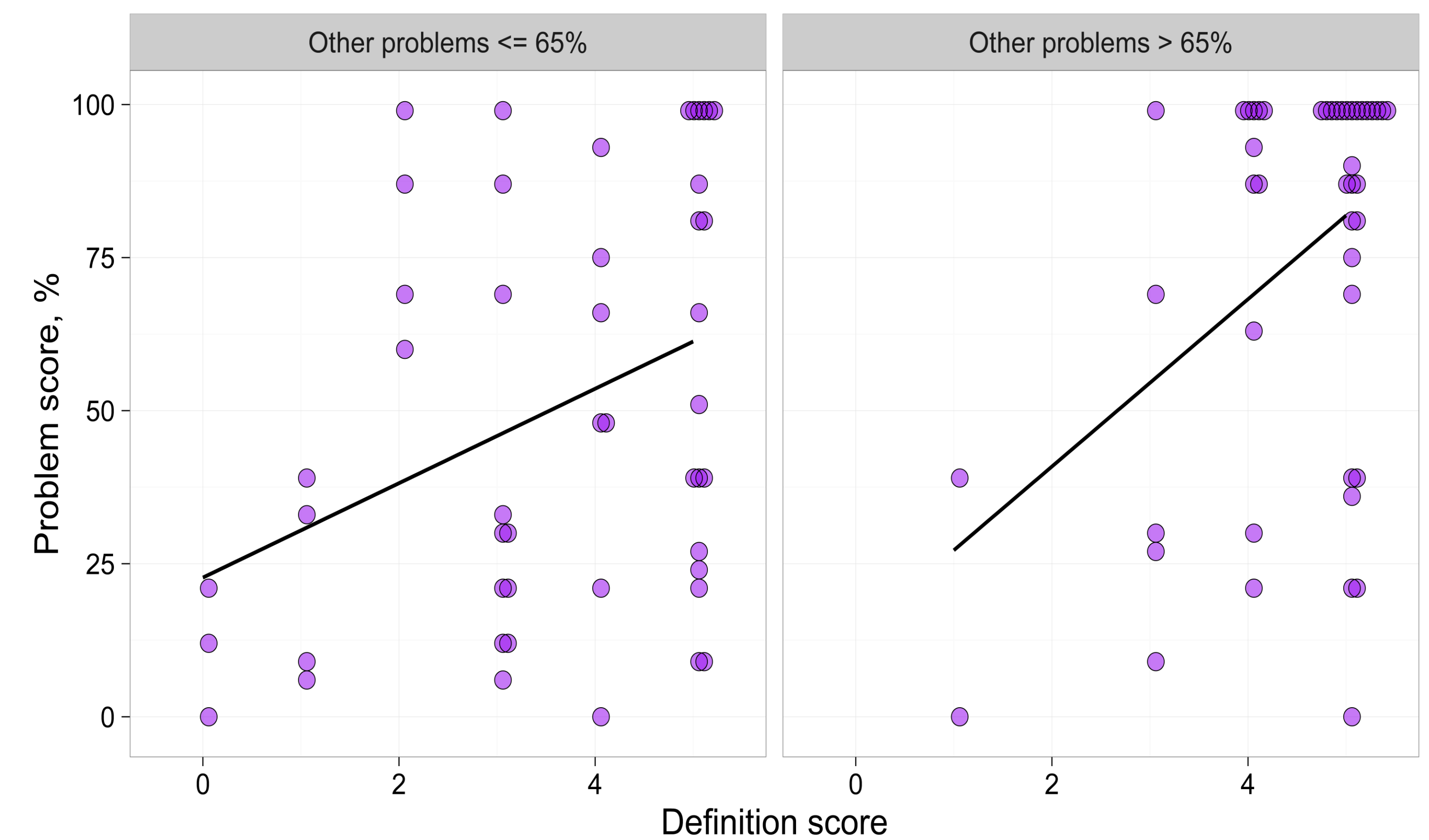## Sample Experiment:

Exam for Control Group:

Problem 3:
Prove that if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
(10 points)

Exam for Experimental Group:

Problem 3:
a. Give a definition of a parallelogram.
(5 points)

b. Prove that if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
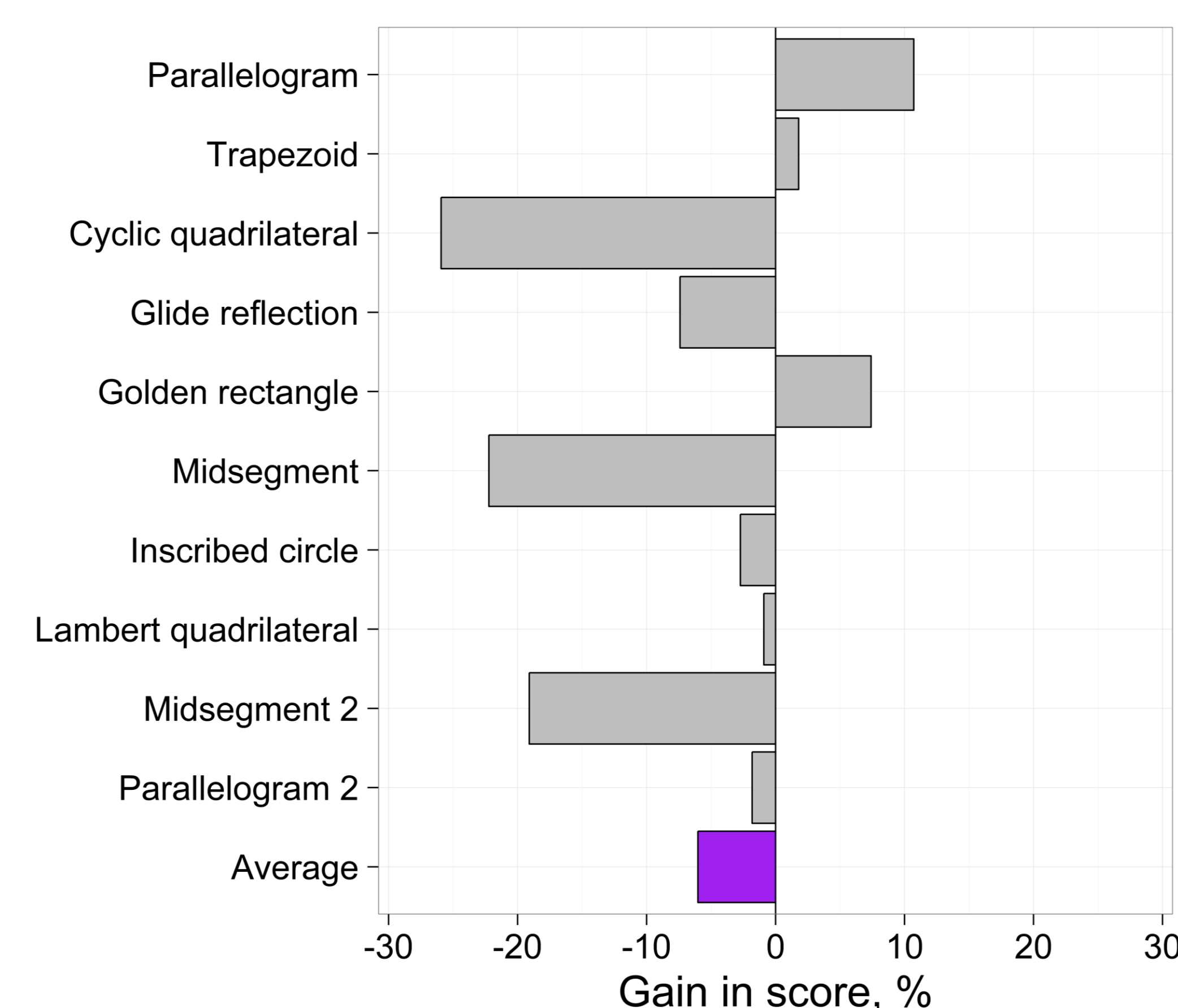*(10 points)*

Results- Question 2

Proof score for the Experimental Group depending on the definition score:

The regression lines indicate higher proof results for the students who scored better on the definitions (6.6%±2.6% per 1 point in the definition score, p= 0.012)

Experimental Design

Details of the study:

- A total of 40 students from two different upper level geometry classes (Math 416 and Math 353) agreed to participate in multiple rounds of experiments which took place from October 2016 to March 2017.

- Overall 108 problem scores were compared.

- Each experiment meant that a randomly assigned half of the class (Control Group) solved a proof problem without any help, while the other half of the class (Experimental Group) was "primed" by having to write out a definition first, related to the problem.

- To make this research project fair to everybody, the two groups were interchanged and assigned to have reverse roles in another problem of the same exam.

- After conducting 10 such experiments, the results were collected in Excel and analyzed with quantitative methods.

- The plots and analyses were performed in R 3.1.2

Results- Question 1

Increase in proof score for the Experimental Group:

Adjusting for the results of other problems, having a definition *decreased* the problem score by 6.6%±4.3%, p=0.13.

Conclusion

From the collected data, it is clear that:

1. **Providing a verbal definition of a related concept before the proof does not help students solve the proof problem.**

2. **Scoring higher on the definitions results in better proof scores, even controlling for results of other problems in the test.**

3. **The knowledge domains of "Verbal Definitions" and "Problem Solving Skills" seem to be completely separated in the geometry students' minds.**

Acknowledgements

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