University of Wisconsin System Math Initiative

Recommendations for Additional Middle Childhood (MC) Mathematics Learning Outcomes

Background:

Mathematics courses for teacher preparation emphasize procedural knowledge, conceptual understanding, and mathematical knowledge for teaching (MKT). MKT develops from experiences with thinking, discussing, reasoning, explaining, justifying, and making sense of mathematical situations. Teacher candidates gain an understanding of how children and adolescents think, common student misconceptions and appropriate interventions, and best practices to support student learning. Increased knowledge in the above areas has been linked to effective teaching, as measured by student achievement. For teacher candidates who wish to teach mathematics in the middle grades, it is imperative that they develop a deep understanding of the mathematical concepts relevant for those grade levels (6-8). Additionally, middle grade teacher candidates must recognize how these mathematical concepts are connected to central ideas in both elementary and high school.

Description:

These are recommendations for additional mathematics course learning outcomes for Middle Childhood teacher candidates interested in teaching middle school mathematics. The following learning outcomes are aligned to content standards typically found in Grades 6-8 with some connections to high school.

The Standards for Mathematical Practice[1] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning, and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[2] when appropriate.

The Number System – Number and Quantity

Appreciate the distinction between properties of operations and rules that are merely conventions about notation (e.g., order of operations).
Apply and extend previous understanding of operations to rational numbers with attention to estimation strategies and visual representations of these operations.
Make sense of properties of rational exponents and the relationship between radicals and rational exponents.
Make sense of the concepts of greatest common factor and least common multiple and apply these concepts to solve mathematical and real world problems.

[1] Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.
National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

[2] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

Explain why decimal expansions of fractions eventually repeat and demonstrate how decimals that eventually repeat can be expressed as fractions.
Explain why irrational numbers are needed and how the number system expands from rational to real numbers.

Algebra

Ratios and Proportional Thinking

Identify, represent, and describe how quantities vary together in a proportional relationship using diagrams[3], tables, equations, and graphs.
Interpret and describe the constant of proportionality as a unit rate in diagrams, tables, equations, and graphs.
Apply proportional relationships to solve ratio and percent problems.
Distinguish proportional relationships from other relationships, such as other linear relationships and inversely proportional relationships.

Expressions and Equations

Distinguish among the roles of symbols as variables, parameters, constants, or functions and attend to precision in algebraic language and notation.
Apply properties of operations to construct equivalent expressions and verify these properties using visual representations.[4]
Understand the concept of a solution set of an equation and inequality[5]; recognize and construct solutions of equations and inequalities using a variety of representations (graphical, numerical/tabular, verbal, algebraic).
Solve a variety of equations[6] and inequalities by applying properties of equality and the field properties of operations to justify the validity of each step and by utilizing visual representations strategically[7].
Recognize processes for solving equations as logical implications that may or may not be biconditional.
Distinguish processes that produce equivalent equations from ones that may yield extraneous solutions.
Create equations and inequalities in one or two variables and use them to solve mathematical and real world problems.
Analyze and solve systems of linear equations graphically, numerically, algebraically, and contextually.
Use similar triangles to recognize the slope is the same between any two distinct points on a non-vertical line and interpret the slope as a constant rate of change.
Derive and interpret various form of linear equations, and describe the roles of the parameters.

[3] Diagrams include tape diagrams and double number lines.
[4] Representations may include tape diagrams, algebra tiles, balance/hanger models.
[5] Inequalities can be used to express a relationship or express restrictions.
[6] Equations include linear and non-linear.
[7] Representations may include tape diagrams, algebra tiles, balance/hanger models.

Functions

Reason about functional relationships and generate a precise definition of a function, including the definitions of domain and range.
Identify functions and nonfunctions represented in different ways including algebraically, graphically, numerically in tables, or by verbal descriptions.
Analyze and compare key features of functional relationships between two variables (how values of the dependent variable can be obtained from values of the independent variable and how change in one variable affects change in the other) represented in different ways including algebraically, graphically, numerically in tables, or by verbal descriptions.
Compare patterns of change across families of functions including linear, quadratic, and exponential functions.
Construct functions to model real world contexts and data.

Geometry

Understand geometry as the study of ideal objects (e.g., angles, circles, polygons) created by their definitions, recognize that doing geometry means reasoning about these ideal objects (as opposed to measuring physical objects), and recognize that precise definitions are necessary for this reasoning as physical or visual representations may be misleading.
Derive the distance formula as an algebraic representation of the Pythagorean Theorem and compute distances between points on a coordinate plane.
Describe the effect of dilations and rigid motions (translations, rotations, reflections) and compositions of these on two-dimensional figures.
Understand and make use of the structure of geometry, stating clear and precise assumptions and definitions in order to construct formal geometric arguments[8] and prove theorems about lines, angles, triangles, and quadrilaterals.
Make geometric constructions using a variety of tools[9] and use formal geometric reasoning[10] to justify that the constructions create what was intended.
Justify the criteria for congruence in terms of rigid motions (translations, rotations, reflections, and compositions of these) and the criteria for similarity in terms of rigid motions and dilations.
Apply congruence and similarity criteria to solve problems and to prove relationships in geometric figures.
Informally justify the formulas for surface area and volume for cylinders, prisms, pyramids, cones, and spheres and apply these formulas to solve mathematical and real world problems.

[8] Formal reasoning does not mean a full axiomatic development; merely that the assumptions used in any individual proof should be clearly spelled out.
[9] Geometric tools may include a straightedge and compass, as well as dynamic geometry software such as GeoGebra, Geometer's Sketchpad, or Desmos.
[10] Formal reasoning does not mean a full axiomatic development; merely that the assumptions used in any individual proof should be clearly spelled out.

Statistics and Probability

In addition to the Standards for Mathematical Practice, the recommendations by the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report[11] should guide the teaching of this content. This should include statistical problem solving as an investigative process using four components: 1) formulate questions, 2) collect data, 3) analyze data, and 4) interpret results.

Use the statistical process to answer questions about bivariate categorical data via two-way frequency tables.
Use the statistical process to answer questions about bivariate quantitative data via scatterplots and linear regression.
Recognize random sampling methods and use data from a random sample to draw inferences about a population.
Critique proposed sampling methods for non-randomness or sources of bias.
Develop and evaluate comparative inferences[12] about two populations using graphs of distributions, measures of center, and measures of spread.
Compute the probabilities of compound events using organized lists, tables, tree diagrams, simulation, and formulas.

[11] The GAISE reports are available at https://www.amstat.org/asa/education/Guidelines-for-Assessment-and-Instruction-in-Statistics-Education-Reports.aspx.

[12] The emphasis should be on statistical reasoning and not on performing statistical tests.

Guidelines for Early Childhood / Middle Childhood Mathematics

In the CBMS report, *Mathematical Education of Teachers II* (MET II), the authors stated the following (CBMS, 2012):

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach (p. 23).
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> As such, these courses provide a foundation for middle school mathematics teaching, but additional coursework would be required for full preparation.

1. The core learning outcomes (LO) were vetted by math and math education faculty systemwide in spring 2020 and serve as a mechanism to ensure consistency for purposes of transfer and applicability of mathematics courses for teachers across the UW System. Individual institutions and faculty will continue to enjoy the freedom to utilize the modality and instructional strategies they deem most appropriate for the delivery of these courses.
2. The Math Education Subcommittee will develop a process to periodically review and update the learning outcomes that honors the autonomy of each department and aligns with state and national standards.
3. The Education Subcommittee recommends a minimum of 6 credits for Early Childhood Education. We believe it is possible to cover the content in this document in 9 credits, but this is insufficient preparation for middle school teacher candidates. We therefore recommend a minimum of 15 credits for teaching middle school mathematics.
4. While this document focuses on mathematical content rather than pedagogy, instructors are strongly encouraged to use the Mathematics Teaching Practices (Principles to Actions, 2014) to guide instructional decisions. It is important for teacher candidates to experience learning mathematics with these practices, and these courses are an opportunity to model meaningful teaching and learning of mathematics. This includes actively engaging students in mathematical reasoning, problem solving, and discourse during class sessions. It also includes using rich tasks and intentionally incorporating student thinking and work in building mathematical understanding. These courses are also a critical opportunity to model the effective use of technology to teach mathematics.

   As such, preservice teachers should be "provided challenging tasks that promote mathematical problem solving" as well as "opportunities to discuss their thinking in small- and full-group discourse" so they can "experience learning mathematics using methods that are consistent with the methods they should use as teachers" (AMTE, 2017, p. 31). These recommendations also include providing pre-service teachers opportunities to analyze task structures so that they can select and create rich mathematical tasks that promote reasoning and problem solving for their future students.

Recommended resources for learning more about teaching using these practices follow.

A Selection of Recommended Resources

- Principles to Actions
- AMTE Standards for Preparing Teachers of Mathematics (https://amte.net/standards)
- The 5 Practices for Orchestrating Productive Math Discussions (2nd ed)
- The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in your Elementary Classroom
- The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in your Middle School Classroom
- *Essential Understandings* texts from NCTM
- Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5
- Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8
- https://www.nctm.org/PtAToolkit/
- Progressions Documents
- *The Mathematics Enthusiast* special issue on supporting mathematics teacher educators' knowledge and practices for teaching content to prospective (grades K-8) teachers, 2020, Vol 17, Numbers 2 and 3

The following are some recommended classroom activities that align closely with the above teaching practices.

- https://www.illustrativemathematics.org/
- https://www.youcubed.org/tasks/
- https://nrich.maths.org/
- 3 Act Tasks (https://gfletchy.com/3-act-lessons)
- https://robertkaplinsky.com/lessons/

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