UNIVERSITY OF WISCONSIN SYSTEM

UW System Math Initiative

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University of Wisconsin System Math Initiative
Early Childhood / Elementary & Middle School Mathematics Learning Outcomes

Note: For teacher candidates wanting to teach middle school mathematics, see guideline 4 below.

Background:

Mathematics courses for teacher preparation emphasize procedural knowledge, conceptual understanding, and mathematical knowledge for teaching (MKT). MKT develops from experiences with thinking, discussing, reasoning, explaining, justifying, and making sense of mathematical situations. Teacher candidates gain an understanding of how children and adolescents think, common student misconceptions and appropriate interventions, and best practices to support student learning. Increased knowledge in the above areas has been linked to effective teaching, as measured by student achievement.

Description:

This course sequence is designed for prospective teachers. Content strands include number and operations, algebraic thinking, geometry, measurement, statistics, probability. This course sequence is aligned with state and national standards.

Whole Numbers and Operations

The Standards for Mathematical Practice[1] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning, and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[2] when appropriate.

Early Childhood (Birth-3rd grade)	Elementary and Middle School (K-9th grade)
Explain essential ideas of counting and cardinality, specifically the role of groups of ten in base-ten place value numeration.	

[1] National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C. The Standards for Mathematical Practice are also outlined in Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.

[2] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

Demonstrate flexible thinking and intuition about numbers[3] and relationships across multiple representations.[4]	
Recognize addition, subtraction, multiplication, and division problem types and associated meanings for the operations.	Recognize addition, subtraction, multiplication, and division problem types and associated meanings for the operations, and that these extend across the real number system.
Operate on whole numbers using place value strategies, mental math, estimation, drawings, and standard and non-standard algorithms.	
Identify, represent, and use the properties (such as commutative, associative, distributive, identity, and inverse) and order of operations.	
	Extend knowledge of whole numbers and operations to rational numbers, integers, and real numbers.
	Make sense of properties of whole number exponents.

Rational Numbers and Operations

The Standards for Mathematical Practice[5] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[6] when appropriate.

Early Childhood (Birth-3rd grade)	**Elementary and Middle School (K-9th grade)**
Flexibly work with a variety of fraction models.[7]	
Recognize fractions as numbers including explaining a fraction *a/b as a part, each of size 1/b*.	
Extend representations of whole number operations to include fraction operations.	
Explain the meaning of fraction operations in mathematical and real-world problems by using visual fraction models, context, and attending to the whole (referent unit).	
	Explain the connection between fractions and division, a/b = a÷b, and how fractions, ratios, and rates are connected via unit rates.
	Interpret and represent rational numbers as decimals.
	Apply and extend operations to positive and negative rational numbers.[8]

[3] Such as number talks.

[4] Such as ten frames and number lines.

[5] National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C. The Standard for Mathematical Practice are also outlined in Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.

[6] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

[7] Models such as length, area, set, and bar.

[8] CCSSM defines a fraction as a positive number.

Algebraic Thinking

The Standards for Mathematical Practice[9] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[10] when appropriate.

Early Childhood (Birth-3rd grade)	Elementary and Middle School (K-9th grade)
Recognize that the equal sign is a symbol that represents a relationship of equivalence, and that equations can be reasoned about in their entirety rather than as a series of computations (e.g. 12 + 38 = 11 + ?).	
Identify and use properties of real numbers (such as commutative, associative, distributive, identity, and inverse) to support sense-making, justification, writing equivalent expressions, and problem solving.[11]	
Identify and explain arithmetic patterns.	Identify and explain patterns to generalize relationships between covarying quantities (proportional, linear, and other functional relationships), expressing those relationships in words, symbols, tables and graphs.[12]
Use mathematical structure, systematic thinking[13] and representations[14] to analyze arithmetic and algebraic situations.	
	Develop an understanding of the symbolic language of algebra, including the role of symbols and the connections of symbols to representations.[15]
	Recognize situations and solve problems that call for proportional reasoning.[16]

[9] National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C. The Standard for Mathematical Practice are also outlined in Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.

[10] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

[11] Such as 22+31 is the same as (20+2)+(30+1)=(20+30)+(2+1) or 17+(-22)=(17+-17)+(-5).

[12] For example, growth pattern problems.

[13] For example, the Mice/Cage Problem (*Jonathan has ten mice in two cages that are joined together. Can you show all the different ways the ten mice could be in the cages?*) (Carpenter, Franke, & Levi, 2003) or the Squirrel/Chipmunk Problem (*For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?*) (IllustrativeMathematics.org)

[14] For example, organized lists, tables, etc.

[15] For example, making sense of situations such as writing an equation to express, "There are three times as many students as teachers."

[16] Representations could include double number lines, tape diagrams, etc.

Geometry and Measurement

Geometry and measurement are closely related. Geometry is the study of idealized versions of objects around us, whereas, measurement allows us to quantify the attributes of real objects so we can make comparisons. The Standards for Mathematical Practice[17] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning, and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[18] when appropriate.

Early Childhood (Birth-3rd grade)	**Elementary and Middle School (K-9th grade)**
Recognize defining attributes of two-dimensional and three-dimensional shapes.	Identify and use mathematical properties of two-dimensional and three-dimensional shapes.
Categorize shapes based on properties. Recognize polygons as a class and distinguish polygons from non-polygons.	
	Use informal arguments to establish facts about the angle sum and exterior angles of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.[19]
	Explain and prove[20] Pythagorean theorem, and apply the theorem to solve problems.
	Explain congruence and similarity using rotations, reflections, translations, and dilations.
Explain and apply the process of measurement: identify the attribute to be measured, choose an appropriate unit, and determine the number of units present.	
Identify length, perimeter, and area as measurable attributes and distinguish among them.	Identify length, perimeter, area, volume, and angle measure as measurable attributes and distinguish among them.
Use estimation (including meaningful benchmarks) to predict the result of a measurement before it is performed and to confirm the reasonableness of the result.	
Use standard and non-standard units of measurement to measure lengths with an appropriate degree of precision.	
	Derive and explain formulas for area of two-dimensional shapes including triangles, special quadrilaterals, and polygons composed of those shapes.

[17] National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C. The Standard for Mathematical Practice are also outlined in Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.

[18] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

[19] These last two theorems are important because they guarantee that the slope of a line is determined by any two points on a line.

[20] A proof of this theorem may include area models.

	Derive and explain formulas for surface area and volume for right cylinders and right prisms. Use informal arguments and physical models to justify volume formulas for cones, pyramids, and spheres. Apply these formulas to both standard and non-standard three-dimensional shapes.
	Give an informal derivation of the relationship between the circumference and area of a circle.[21] Use these formulas to solve problems.
	Convert between measurement units[22] using strategies that develop reasoning and understanding.
	Explain the effects of dilations on measurable attributes of shapes such as length, area, and volume.

Probability and Statistics

The GAISE Report[23] should also guide the teaching of the content in this domain with an emphasis on the statistical process of 1) formulating a question, 2) collecting data, 3) analyzing data, and 4) interpreting results. Connections should be made to contextual problems and representations when appropriate. The Standards for Mathematical Practice[24] should guide the teaching of the content in this domain with a focus on conceptual understanding, procedural fluency, mathematical reasoning, and problem solving. Connections should be made to representations and real world problems as defined by authentic contextual problems through mathematical modeling[25] when appropriate.

Early Childhood (Birth-3rd grade)	**Elementary and Middle School (K-9th grade)**
Understand concepts of probability and chance and informally evaluate likelihood of events and develop the idea of probability as long run behavior.	
	Calculate theoretical probabilities using probability models[26] and experimental probabilities[27] and compare these.
Recognize and describe distinguishing characteristics of a meaningful statistical question.	

[21] Derive the area formula from the circumference and/or circumference from the area formula.

[22] Within measurement systems and across measurement systems.

[23] Detailed Guidelines for Assessment and Instruction in Statistics Education can be found at https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf.

[24] National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C. The Standard for Mathematical Practice are also outlined in Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM, National Council of Teachers of Mathematics, 2014.

[25] As defined by Common Core State Standards for Mathematics: "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, D.C.

[26] Using tools such as organized lists, tables, tree diagrams, Venn diagrams and area models.

[27] Using the Fundamental Principle of Counting.

Collect categorical and numerical data using appropriate data collection methods to reduce bias.	
Construct appropriate visual representations of data such as pictographs, bar graphs, and line plots[28] to answer statistical questions.[29]	Construct appropriate visual representations of data such as pictographs, bar graphs, dot plots, histograms, box plots, and scatterplots to answer statistical questions.[30]
Explain and interpret the meaning of the mode with reference to the context in which data was gathered.	Explain and interpret[31] measures of center (mean and median) and spread (range, interquartile range, and mean absolute deviation) with reference to the context in which data was gathered.
	Describe statistical variability and the role of randomness in statistical inference.

[28] CCSSM defines line plot in Grades K-5 as method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. It is known as a dot plot in Grades 6-8.

[29] This should include emphasis on how visual representations and statistics can be used to mislead.

[30] This should include emphasis on how visual representations and statistics can be used to mislead.

[31] Interpret results with awareness of distinction between conclusions (relate back to original context).

Guidelines for Early Childhood / Middle Childhood Mathematics

In the CBMS report, *Mathematical Education of Teachers II* (MET II), the authors stated the following (CBMS, 2012):

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach (p. 23).
>
> As such, these courses provide a foundation for middle school mathematics teaching, but additional coursework would be required for full preparation.

1. The core learning outcomes (LO) were vetted by math and math education faculty systemwide in spring 2020 and serve as a mechanism to ensure consistency for purposes of transfer and applicability of mathematics courses for teachers across the UW System. Individual institutions and faculty will continue to enjoy the freedom to utilize the modality and instructional strategies they deem most appropriate for the delivery of these courses.
2. The Math Education Subcommittee will develop a process to periodically review and update the learning outcomes that honors the autonomy of each department and aligns with state and national standards.
3. One intent of the learning outcomes is that—if a student successfully completes a sequence of courses across a comparable number of credits meeting the learning outcomes at one UW institution and transfers the sequence to another—the receiving institution may accept the sequences of courses for credit, as meeting an existing mathematics-related graduation requirement.
4. The Education Subcommittee recommends a minimum of 6 credits for Early Childhood Education. We believe it is possible to cover the content in this document in 9 credits, but this is insufficient preparation for middle school teacher candidates. We therefore recommend a minimum of 15 credits for teaching middle school mathematics. ***Recommendations for Additional Middle Childhood Mathematics Learning Outcomes*** have been developed related to additional teacher preparation of mathematics courses for those interested in teaching middle school.
5. While this document focuses on mathematical content rather than pedagogy, instructors are strongly encouraged to use the Mathematics Teaching Practices (Principles to Actions, 2014) to guide instructional decisions. It is important for teacher candidates to experience learning mathematics with these practices, and these courses are an opportunity to model meaningful teaching and learning of mathematics. This includes actively engaging students in mathematical reasoning, problem solving, and discourse during class sessions. It also includes using rich tasks and intentionally incorporating student thinking and work in building mathematical understanding. These courses are also a critical opportunity to model the effective use of technology to teach mathematics. Recommended resources for learning more about teaching using these practices follow.

   As such, preservice teachers should be "provided challenging tasks that promote mathematical problem solving" as well as "opportunities to discuss their thinking in small- and full-group discourse"

so they can "experience learning mathematics using methods that are consistent with the methods they should use as teachers" (AMTE, 2017, p. 31). These recommendations also include providing pre-service teachers opportunities to analyze task structures so that they can select and create rich mathematical tasks that promote reasoning and problem solving for their future students.

A Selection of Recommended Resources

- Principles to Actions
- AMTE Standards for Preparing Teachers of Mathematics (https://amte.net/standards)
- The 5 Practices for Orchestrating Productive Math Discussions (2nd ed)
- The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in your Elementary Classroom
- The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in your Middle School Classroom
- *Essential Understandings* texts from NCTM
- Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5
- Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8
- https://www.nctm.org/PtAToolkit/
- Progressions Documents
- *The Mathematics Enthusiast* special issue on supporting mathematics teacher educators' knowledge and practices for teaching content to prospective (grades K-8) teachers, 2020, Vol 17, Numbers 2 and 3.

The following are some recommended classroom activities that align closely with the above teaching practices.

- https://www.illustrativemathematics.org/
- https://www.youcubed.org/tasks/
- https://nrich.maths.org/
- 3 Act Tasks (https://gfletchy.com/3-act-lessons)
- https://robertkaplinsky.com/lessons/

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