Statement about College Readiness and the Wisconsin Standards for Mathematics from Wisconsin Higher Education and K-12 Representatives

As Wisconsin's K-12 leaders and higher-education mathematics faculty, we have created this document to outline the necessary mathematical knowledge, skills, and behaviors for students planning to attend Wisconsin postsecondary education. We believe these attributes are universally necessary across higher education and are fostered by the updated Wisconsin Standards for Mathematics (WSM, 2021). In short, the new WSM, when properly implemented, will prepare Wisconsin's students to succeed.

Mathematical readiness for higher education is about understanding mathematics, in depth and in context, and consists of much more than procedures and content (though they are important, too). Because of this, students need to have developed critical mathematical habits of mind–mathematical curiosity, perseverance, creativity, autonomy, and collaboration–through ongoing experiences in problem solving, communicating, making arguments, and thinking critically. Below, we discuss four aspects of the WSM that are of particular importance to this wider view of postsecondary readiness: (1) the focus on standards for mathematical practices (WSM, 2021, pp. 27-28), (2) the coherence of content standards across grades and content areas, (3) the interwoven theme of mathematical modeling across the grades, and (4) the options for alternative mathematical pathways offered in the last two years of high school.

(1) The Focus on Standards for Mathematical Practices (SMPs): Postsecondary faculty value the critical mathematical habits of mind listed above, and count on students to make sense of mathematical ideas and understand what it means to *do mathematics*. The SMPs describe how students at all levels should develop these habits, and we applaud their prominence in the WSM. After all, these are the practices used not just by working mathematicians, but by many people in a wide variety of careers, besides their ubiquity in daily life. Moreover, the ways of thinking described by the SMPs will remain long after specific formulas, definitions, or procedures are forgotten. These practices should be addressed on a regular basis in every grade. For example, young children can make conjectures about the structure of arithmetic (Carpenter, Franke, and Levi, 2003): they can note that when you add zero to a number you get the same number back, or argue by using arrays that three groups of four objects are the same number as four groups of three objects. Elementary teachers may use number talks to encourage children to solve problems mentally and explain their thinking. In higher grades, teachers can build on these foundations to help their students develop number sense and quantitative reasoning skills. These skills are valued by higher education faculty, at both four year and technical colleges. The SMPs require teachers to treat all students as capable and valued participants, engage them with high quality mathematical ideas and challenging problems, and promote independent thinking and autonomy. As such, the SMPs promote the new Shifts #1 and #2 (WSM, 2021, p. 10) in the WSM of valuing student identity and seeing students as flexible users of mathematics. The SMPs envision the classroom as a place where *all* students have the opportunity to *understand* mathematics.

(2) The Coherence of Content Standards Across Grades and Content Areas: Knowledge of skills and procedures is an important part of mathematical readiness for higher education. However, this knowledge must go deeper than mechanical manipulation of symbols: Students must "see through" the symbols and understand the concepts those symbols represent. This deep understanding of mathematical content is enabled particularly by Shift #4: "All students engage in coherent mathematics that connects concepts and mathematical thinking within and across domains and grades" (WSM, 2021, p. 11). One important example of this coherence in the WSM is the careful and intentional development of knowledge of fractions in Grades 3 through 6. When districts attend to this progression in an implemented curriculum, students will learn far more than rote algorithms for adding or multiplying fractions (algorithms that are easily forgotten or misapplied). They also will come to understand the meaning of fractions as quantities and see the meaning of the arithmetic operations with fractions as identical to those for natural numbers. This understanding of the meaning of operations is essential for future success in algebra and other advanced courses (including calculus). As an example of coherence across domains, the suggested theorems for which students are asked to give informal arguments in the Grade 8 Geometry domain (M.8.G.A.5) are precisely those that are necessary to explain the concept of slope of a line in Grade 8 Expressions and Equations (M.8.EE.B.5). Coherence in the WSM extends into the high school standards also, not least in the continuity of expectations between middle and high school. Recognizing the coherence in the WSM and using it to guide the implemented curriculum will result in all students working on grade-level mathematics, and proceeding to the study of more advanced content with greater depth of understanding, as required for higher education.

(3) The Interwoven Theme of Mathematical Modeling Across the Grades: Facility with mathematical modeling is important for the study of biology, chemistry, physics, economics, sports medicine, game theory, and a host of college and technical college subjects (and, of course, it also is vital in the workplace). The WSM call for mathematical modeling to be used "throughout K-12 mathematical experiences that allow for exploration of authentic math problems that arise in everyday lives to support each student's identity as a problem solver" (p. 18). In responding to this call, it is important to recognize that mathematical modeling is more than simply asking students to solve word problems in context. It is also more than just using objects such as manipulatives to represent mathematical ideas. Rather, "Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions" (GAIMME Report, 2016). Answering big, messy problems requires students to work through an iterative process of understanding the problem, analyzing the situation, creating a mathematical model, and analyzing and assessing the model (WSM Appendix 4, 2021, p. 224). Rather than being presented with specific questions, students themselves should be asking at the beginning of the modeling process, "What questions need answering?" and "What information is needed?" (WSM, 2021, p. 224), as well as "What mathematical tools should we use and why?".

The openness of the modeling process implies that a diversity of perspectives is crucial, and students should be encouraged to build their own mathematical identities. These ideas are stressed in the new Shifts #1 and #2 of the WSM (2021, p. 10) and are strongly aligned with Wisconsin's Guiding Principles for Teaching and Learning (WSM, 2021, p. 2). The ability to work through the full modeling process is not innate; it should therefore be intentionally developed throughout the K-12 curriculum, even in the early grades. Indeed, the WSM description of the Mathematical Modeling Practice Standard for K-5 states "Mathematically proficient elementary students formulate their own problems that emerge from natural circumstances as they mathematicise the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics" (WSM, 2021, p. 26). In essence, K-5 students should be able to use the modeling process as a way to paint *their* world and *their* experiences with mathematics using tools of *their invention* and innovation. In answering the questions they generate, students should bring to the problem(s) any appropriate mathematical tools they know. In higher grades, modeling therefore has connections to geometry, statistics, and computer science, not merely to algebra and calculus. Appendix 4 of the WSM provides more detail on how mathematical modeling should develop across the grades; careful attention to that appendix, and to the other expectations for mathematical modeling in the WSM, will enable students to see mathematics as useful and doable in their future studies, careers, and lives.

(4) The Options for Alternative Mathematical Pathways Offered in the Last Two Years of High School: We turn finally to the question of alternative mathematical pathways. Without attempting to impose a "one size fits all" approach to the high school curriculum, we believe the challenge here is to steer the narrow path between inappropriate acceleration (the excessively early introduction of algebra and calculus), which all too often results in an over-emphasis on skills and procedures over the mathematical practices, and a premature opening of alternative pathways, which is likely to narrow students' future options in higher education. Accordingly, alternative pathways are never appropriate in Grades 9 and 10, and rarely advisable for college-bound students even in Grade 11, while it is entirely appropriate for a student to choose a pathway appropriate for their major once they are in college and have decided on that major.

We stress the importance of a deep understanding of the core branches of mathematics– algebra and geometry (and, increasingly, statistics)–for students' postsecondary preparation. Increasingly, college faculty prefer that incoming students have a strong foundation in mathematics prior to calculus and are persistent problem-solvers. A solid background in algebra, in particular, is needed not only by STEM majors, but by majors in many other fields, such as Business, and for this reason many four-year colleges and universities still expect students to take a significant amount of algebra in the third and fourth years of high school. The importance of quantitative reasoning, number sense, mathematical modeling, and statistics cannot be overrated, but for precisely that reason, these topics and skills should be embedded in the core curriculum and not relegated to alternative pathways. These elements should not be sacrificed in favor of acceleration to or through calculus, and only a very small number of truly exceptional students will benefit from arriving as first-year college students with credit beyond the first year of calculus.

Summary: We believe the Wisconsin Standards for Mathematics, implemented with integrity, provide opportunities for Wisconsin's future citizens to become powerful mathematical thinkers, collaborative and creative humans, engaged and thoughtful participants in society, and valued and productive employees. We hope this document clearly communicates our hopes and expectations for postsecondary preparation of those future citizens and our belief that the WSM promotes those expectations.

References

Wisconsin Department of Public Instruction. (2021) *Wisconsin Standards for Mathematics (WSM).* Retrieved from https://dpi.wi.gov/sites/default/files/imce/standards/New%20pdfs/MathematicsStandards2021.pdf.

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Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME, 2016). 7-22. United States of America: Consortium for Mathematics and Its Applications (COMAP, Inc.) and Society for Industrial and Applied Mathematics (SIAM).

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Supplemental materials:

UW System Mathematics Placement Test (MPT) – The MPT is designed to place students into the course, ranging from fundamental mathematics to first semester calculus, that is most appropriate for them to begin their college-level mathematics study and have the greatest opportunity to succeed. Because the relevant courses are most often algebra courses (or have algebra courses as prerequisites), the MPT emphasizes algebraic skills. Despite this, the MPT is not intended to provide a summative assessment of a student's high school mathematics learning. The sole purpose of this test is to determine how well-prepared students are to begin taking mathematics courses at a University of Wisconsin campus. Therefore, the test is designed with University of Wisconsin System curriculum in mind. The MPT also addresses mathematical practices and reasoning skills: many problems can be solved quickly by students with good number sense or reasoning skills, where a reliance on memorized procedures would take much longer. In any case, a rich set of mathematical experiences in high school will prepare students for the MPT far better than an over-emphasis on calculation and procedures.

UW System Early Math Placement Tool (EMPT) - The EMPT is a program sponsored by the UW-System, the Wisconsin Technical College System, and the WI Department of Public Instruction, in partnership with Wisconsin High Schools, that is designed to provide high school students with information on their preparation for pursuing math courses at the postsecondary level. The EMPT is a shortened version (40 items) of the UW System Mathematics Placement Test, using items retired from that test. Upon completing the assessment, students will receive a placement level. The online resources can be used to learn what college courses correspond to each placement level at each of the UW System campuses and WTCS campuses. Information on math requirements by major on each campus is also provided. This program is available free of charge to all Wisconsin high schools.

The Cold Hard Truth about School Math –TED Talk by Jen Szydlik
School math is a ruthless and ineffectual substitute for the real thing. Students often experience "math as a race" where quick memorization of tools pays dividends and powerful and authentic mathematical behaviors are marginalized–along with the students themselves.

Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations
Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations
Catalyzing Change in High School Mathematics: Initiating Critical Conversations Resources for educators who are looking for the position of the National Council of Teachers of Mathematics regarding issues similar to those in this position statement

Data Science and the High School Math Curriculum A statement by academic staff at California 4-year colleges and universities

Invigorating High School Math: Practical Guidance for Long-Overdue Transformation High school math is failing many students. Out-of-date and stale curricula are not only dull but perpetuate inequity by limiting opportunities and failing to prepare a majority of students for life in the 21st century. Invigorating High School Math is a clarion call for meaningful transformation. In this book, Steven Leinwand and Eric Milou address the most critical challenges facing high school mathematics and provide practical guidance for meeting them.

Bargagliotti, Anna and Christine Franklin. 2020. Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education. Alexandria: American Statistical Association.